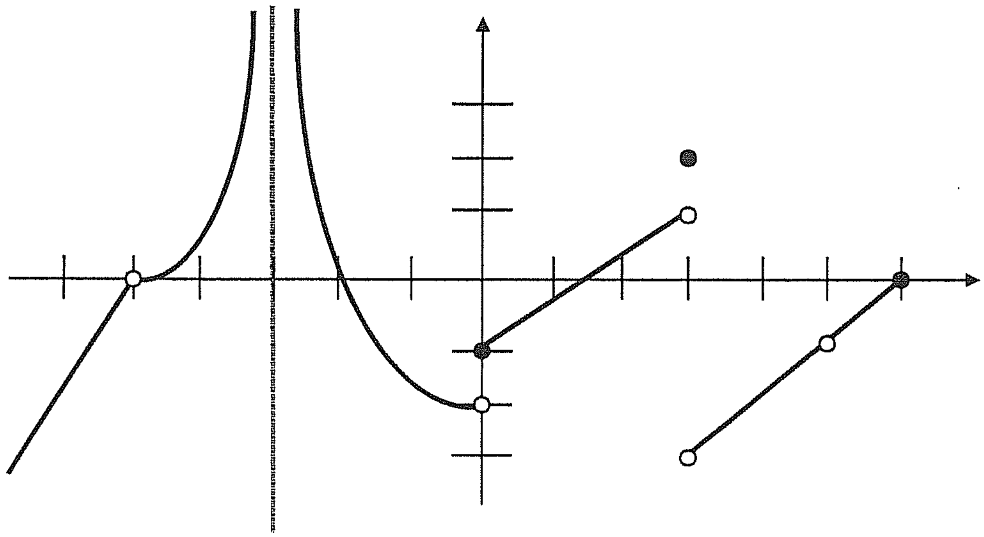
Name______________________________

AP Calculus BC Summer Review Packet (Limits & Derivatives)

Due First day back

Limits

1. **Answer the following questions using the graph of $f(x)$ given below.**



(a) Find $f(0)$

(b) Find $f(3)$

(c) Find $\lim_{x\to -5} f(x)$

(d) Find $\lim_{x\to 0^-} f(x)$

(e) Find $\lim_{x\to 3^-} f(x)$

(f) Find $\lim_{x\to -3^+} f(x)$

(g) List all x-values for which $f(x)$ has a removable discontinuity. Explain what section(s) of the definition of continuity is (are) violated at these points.

(h) List all x-values for which $f(x)$ has a nonremovable discontinuity. Explain what section(s) of the definition of continuity is (are) violated at these points.

In problems 2-10, find the limit (if it exists) using analytic methods (i.e. without using a calculator).

2. $\lim_{x \to -2} \frac{3x^2 + 21x + 30}{x^3 + 8}$

3. $\lim_{x \to \pi/6} \frac{1 - \cos^2 x}{4x}$

4. $\lim_{x \to 4} \frac{\sqrt{x-3} - 1}{x - 4}$

5. $\lim_{x \to 0} \frac{1/(x+1) - 1}{x}$

6. $\lim_{x \to 0} \frac{\left[1/\sqrt{1+x}\right]-1}{x}$

7. $\lim_{\theta \to 0} \frac{\sin 6\theta^3}{7\theta}$

8. $\lim_{t \to 0} \frac{\sin^2 3t^2}{t^3}$

9. $\lim_{x \to 6^-} \frac{|6x-36|}{6-x}$

10. $\lim_{\Delta x \to 0} \frac{\sin((\pi/6)+\Delta x)-(1/2)}{\Delta x}$

Hint: $\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

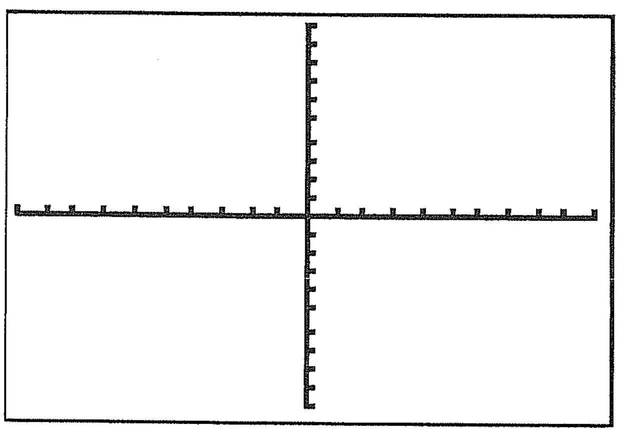
11. **Suppose** $f(x)=\begin{cases}\dfrac{\sqrt{2x+1}-\sqrt{3}}{x-1}, x\geq 0\\ 4x^2+k, x<0\end{cases}$.

(a) For what value of *k* will *f* be piecewise continuous at *x* = 0? Explain why this is true using one-sided limits. (**Hint: A function is continuous at**

***x* =*c* if (1) *f*(*c*) exists, (2)** $\lim_{x\to c} f(x)$ **exists, and (3)** $\lim_{x\to c} f(x)=f(c)$ **.)**

(b) Using the value of *k* that you found in part (a), **accurately** graph *f* below. Approximate the value of $\lim_{x\to 1} f(x)$

$\lim_{x\to 1} f(x)=$ ________________



(c) Rationalize the numerator to find $\lim_{x\to 1} f(x)$ analytically.

12. **Analytically** determine the values of *b* and *c* such that the function *f* is continuous on the entire real number line. **See the hint given in problem 11.**

$$f(x)=\begin{cases} x+1, 1<x<3 \\ x^2+bx+c, x<1 \text{ or } x>3 \end{cases}$$

In problem 13, circle the correct answer and explain why the answer is the correct one.

13. If $f(x)=x^3+x-3$, and if *c* is the only real number such that *f(c)* = 0, then by the Intermediate Value Theorem, *c* is necessarily between

(A) -2 and -1

(B) -1 and 0

(C) 0 and 1

(D) 1 and 2

(E) 2 and 3

Hint: The Intermediate Value Theorem states that if *f* is a continuous function on the interval [a, b] and *k* is any number between *f(a)* and *f(b)*, then there must exist at least one number $c \in [a,b]$ such that *f(c)* = *k*.

Derivatives

In problems 1 & 2, find the derivative of the function by using the limit definition of the derivative.

1. $f(x) = x^3 - 2x + 3$

2. $f(x) = \dfrac{x+1}{x-1}$

In problems 3-14, find the derivative of the given function using the power, product, quotient, and/or chain rules.

3. $f(x) = (3x^2 + 7)(x^2 - 2x + 3)$

4. $f(x) = \sqrt{x}\sin x$

5. $f(t) = t^3 \cos t$

6. $f(x) = \dfrac{x^2 + x - 1}{x^2 - 1}$

7. $f(x) = \dfrac{x^4 + x}{\tan^2 x}$

8. $f(x) = 3x^2 \sec^3 x$

9. $f(x) = 3x \csc x + x \cot x$

10. $f(x) = \left(\dfrac{x+5}{x^2 - 6x}\right)^2$

11. $f(x)=(x^3-2)^{3/2}(5x^2+1)^{5/2}$

12. $f(x)=x^3\cot^4(7x)$

13. $f(x)=5\sin^2(\sqrt{3x^4+1})$

Problems continue on the next page.

In problems 14 & 15, find an equation of the tangent line to the graph of *f* at the indicated point *P*.

14. $f(x) = \dfrac{1+\cos x}{1-\cos x}, P\left(\dfrac{\pi}{2}, 1\right)$

15. $f(x) = \ x^2 - 1 \ ^{2/3}, P(3,4)$

In problems 16 & 17, find the second derivative of the given function.

16. $f(x) = (4x^2 - 3x)^{3/2}$

17. $h(x) = x^3 \cos(\pi x)$

In problem 18, use the position function $s(t) = -16t^2 + v_0 t + s_0$ **for free-falling objects.**

18. A ball is thrown straight down from the top of a 220-foot tall building with an initial velocity of -22 feet per second.

(a) Determine the average velocity of the ball on the interval [1, 2].

(b) Determine the instantaneous velocity of the ball at $t = 3$.

(c) Determine the time t at which the average velocity on [0, 2] equals the instantaneous velocity.

(d) What is the velocity of the ball when it strikes the ground?

In problem 19-24, circle the correct answer and explain why the answer is the correct one.

19. $$\lim_{h \to 0} \frac{\cos\left(\frac{\pi}{6}+h\right)-\cos\left(\frac{\pi}{6}\right)}{h} =$$

(A) Does not exist

(B) $\frac{1}{2}$

(C) $-\frac{1}{2}$

(D) $\frac{\sqrt{3}}{2}$

(E) $-\frac{\sqrt{3}}{2}$

20. Let *f* and *g* be differentiable functions with values for *f(x), g(x), f'(x), and g'(x)* shown below for *x* = 1 and *x* = 2.

x	*f(x)*	*g(x)*	*f'(x)*	*g'(x)*
1	4	-4	12	-8
2	5	1	-6	4

Find the value of the derivative of $f(x) \bullet g(x)$ at *x* = 1.

(A) -96

(B) -80

(C) -48

(D) -32

(E) 0

21. Let $f(x) = \begin{cases} 3x^2 + 4, x < 1 \\ x^3 + 3x, x \geq 1 \end{cases}$. Which of the following is true?

I. *f(x)* is continuous at *x* = 1

II. *f(x)* is differentiable at *x* = 1

III. $\lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x)$

(A) I only

(B) II only

(C) III only

(D) I and III only

(E) II and III only

22. The equation of the line tangent to the curve $f(x)=\frac{kx+8}{k+x}$ at x = -2 is $y = x + 4$. What is the value of k?

(A) -3

(B) -1

(C) 1

(D) 3

(E) 4

23. An equation of the line <u>normal</u> to the curve $y=\sqrt[3]{x^2-1}$ at the point where x = 3 is

(A) $y+12x=38$

(B) $y-4x=10$

(C) $y+2x=4$

(D) $y+2x=8$

(E) $y-2x=-4$

Hint: A <u>normal line</u> to a curve at a point is perpendicular to the tangent line to the curve at the same point.

24. If the nth derivative of y is denoted as $y^{(n)}$ and $y = -\sin x$, then $y^{(14)}$ is the same as

(A) y

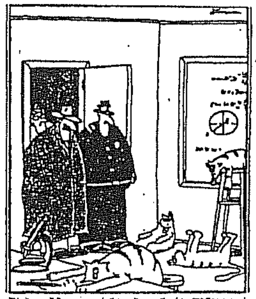
(B) $\frac{dy}{dx}$

(C) $\frac{d^2y}{dx^2}$

(D) $\frac{d^3y}{dx^3}$

(E) None of the above

Related Rates



"Notice all the computations, theoretical scribblings, and lab equipment, Norm. ... Yes, curiosity killed these cats."

Names ______________________

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(1) A stone dropped into a still pond sends out a circular ripple whose radius increases at a constant rate of 3 ft/sec. How rapidly is the area enclosed by the ripple increasing at the end of 10 sec?

(1) ______________

(2) A 17-ft ladder is leaning against a wall. If the bottom of the ladder is pulled along the ground away from the wall at a constant rate of 5 ft/sec, how fast will the top of the ladder be moving down the wall when it is 8 ft above the ground?

(2) ______________

(3) A boat is pulled into a dock by means of a rope attached to a pulley on the dock (Figure 4.1.10). The rope is attached to the bow of the boat at a point 10 ft below the pulley. If the rope is pulled through the pulley at a rate of 20 ft/min, at what rate will the boat be approaching the dock when 125 ft of rope is out?

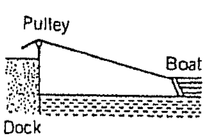

Figure 4.1.10

(3)

④ At a certain instant each edge of a cube is 5 in. long and the volume is increasing at the rate of 2 in^3/min. How fast is the surface area of the cube increasing?

④ ______________

⑤ An aircraft is climbing at a 30° angle to the horizontal. How fast is the aircraft gaining altitude if its speed is 500 mi/hr?

⑤ ______________

⑥ A particle is moving along the curve whose equation is

$$\frac{xy^3}{1+y^2} = \frac{8}{5}$$

Assume that the x-coordinate is increasing at the rate of 6 units/sec when the particle is at the point (1, 2).

(a) At what rate is the y-coordinate of the point changing at that instant?

(b) Is the particle rising or falling at that instant?

⑥

Answers

Limits:

1. (a) -1

   (b) 2

   (c) 0

   (d) -1

   (e) 1

   (f) $+\infty$

   (g) x = -5, 5

   (h) x = -3, 0, 3

2. 3/4

3. 3/(8π)

4. 1/2

5. -1

6. -1/2

7. 0

8. 0

9. 6

10. $\frac{\sqrt{3}}{2}$

11. (a) $k = -1 + \sqrt{3}$

    (b) $\approx .577$

    (c) $\frac{1}{\sqrt{3}}$

12. b = -3, c = 4

13. D

Derivatives:

1. $f'(x) = 3x^2 - 2$

2. $f'(x) = \frac{-2}{(x-1)^2}$

3. $f'(x) = 12x^3 - 18x^2 + 32x - 14$

4. $f'(x) = \sqrt{x}\cos x + \frac{\sin x}{2\sqrt{x}}$

5. $f'(t) = -t^3 \sin t + 3t^2 \cos t$

6. $f'(x) = \frac{-x^2 - 1}{(x^2-1)^2}$

7. $f'(x) = \frac{4x^3 \tan x + \tan x - 2x^4 \sec^2 x - 2x \sec^2 x}{\tan^3 x}$

8. $f'(x) = 9x^2 \sec^3 x \tan x + 6x \sec^3 x$

9. $f'(x) = -3x \csc x \cot x + 3\csc x - x\csc^2 x + \cot x$

10. $f'(x)=\dfrac{(2x-10)(-x^2-10x-30)}{(x^2-6x)^3}$

11. $f'(x)=25x((x^3-2)(5x^2+1))^{3/2}+\dfrac{9}{2}x^2(5x+1)^{5/2}(x^3-2)^{1/2}$

12. $f'(x)=-28x^3\cot^3(7x)\csc^2(7x)+3x^2\cot^4(7x)$

13. $f'(x)=\dfrac{60x^3\sin\sqrt{3x^4+1}\cos\sqrt{3x^4+1}}{\sqrt{3x^4+1}}$

14. $y-1=-2(x-\frac{\pi}{2})$

15. $y-4=2(x-3)$

16. $f'(x)=12\sqrt{4x^3-3x}+\dfrac{3(8x-3)^2}{4\sqrt{4x^2-3x}}$

17. $h'(x)=-\pi^2x^3\cos\pi x-6\pi x^2\sin\pi x+6x\cos\pi x$

18. (a) -70 ft./s.

(b) -118 ft./s.

(c) t = 1 s.

(d) ≈ -120.688 ft./s.

19. C

20. B

21. B

22. D

23. D

24. C

Answers to Related Rates

1. 180 π ft^2/sec
2. -9.375 ft/sec
3. -20.064 ft/min
4. 1.6 in^2/min
5. 250 mi/hr
6. 8.571, falling